Competing instabilities of rotating boundary-layer flows in an axial free-stream

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Introduction

In this study, a new centrifugal instability mode, which dominates within the boundary-layer flow over a slender rotating cone, defined by half-angle $\psi < 40°$, is used for the first time to model the problem when an enforced oncoming axial flow is introduced. The resulting similarity solution represents the basic flow more accurately than previous studies in the literature. This mean flow field is subsequently perturbed leading to disturbance equations that are solved via numerical and analytical approaches, importantly yielding favourable comparison with existing experiments. Meanwhile, a formulation consistent with the classic rotating-disk problem has been successful in predicting the stability characteristics of broad rotating cones, defined by half-angle $\psi > 40°$, in axial flow.

Physically, the problem represents an accurate model of axial airflow over rotating machinery components at the leading edge of a turbofan. In such applications, laminar-turbulent transition within the boundary layer can lead to significant increases in drag, resulting in negative implications for fuel efficiency, energy consumption and noise generation. Consequently, delaying transition to turbulent flow is seen as beneficial, and controlling the primary instability may be one route to achieving this. Ultimately, control of the input parameters of such a problem may lead to future design modifications and potential cost savings.

Our results are discussed in terms of existing experimental data and previous stability analyses on related bodies. Importantly, axial flow is seen to delay the onset of convective instability for both broad and slender rotating cones; the exact mechanism of interaction governing the transition process however is very different for both instabilities. Broad-angled rotating cones are susceptible to a crossflow instability visualised in terms of co-rotating spiral vortices, whereas slender rotating cones have transition characteristics governed by a centrifugal instability, which is visualised by the appearance of counter-rotating Görtler vortices. It is the relative competition of these two governing mechanisms that is explored in detail in this study, particularly with regard to the role of travelling modes in the breakdown process.

Keywords: Rotating boundary-layer; crossflow instability; centrifugal instability; broad/slender rotating cone; co-rotating/counter-rotating spiral vortices.

1. Methods

We consider a cone of half-angle ψ rotating in a fluid of kinematic viscosity ν^* with an angular velocity Ω^* in an anti-clockwise direction around the streamwise coordinate axis x^* (where a * denotes a dimensional quantity in all that follows). We construct coordinate axes aligned along with and perpendicular to the spiral vortices ($\hat{x}^*$ and y^*, respectively), as shown in Figure 1. These are shifted from the conventional streamwise and azimuthal coordinates, x^* and θ, which are based on cylindrical polar coordinates. With the important distinction of the inclusion of the oncoming axial flow, the physical problem is subsequently altered such that there now exists a dimensional local slip velocity at the edge of the boundary layer, obtained via a well-known potential-flow solution (see for example [1]),

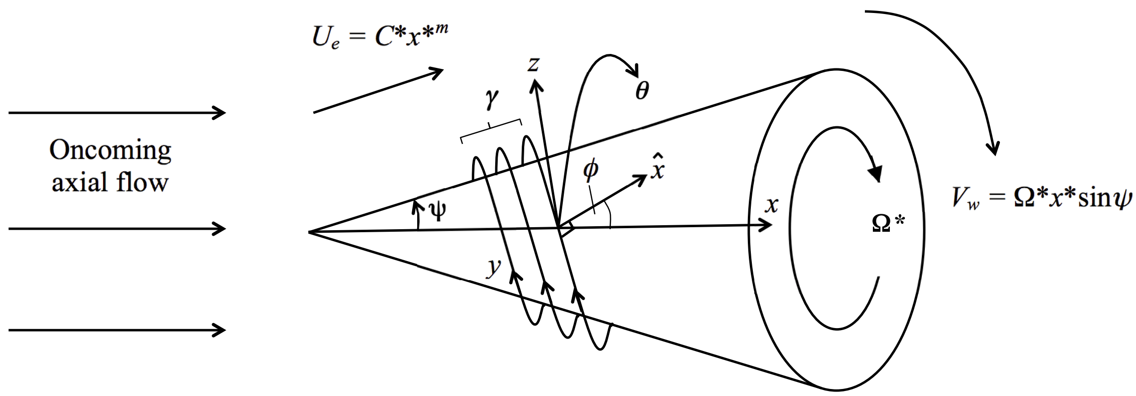

Figure 1. Diagram of the spiral vortex instability on a rotating cone placed in an oncoming axial flow, showing coordinates in the $\hat{x}$- and y-logarithmic spiral directions, as well as the corresponding vortex wavenumber, γ, and vortex waveangle, ϕ.

given by $U_e = C^* x^{*m}$, where C^* is a constant.

We subsequently compare this velocity to the rotational velocity of the cone surface, given by $V_w = \Omega^* x^* \sin\psi$, to obtain an important ratio, which in part characterises the problem, known as the rotational-flow parameter, given by

$$s = \left(\frac{V_w}{U_e}\right)^2,$$

and used in [2, 3, 4]. In this study, we will use s to facilitate comparison of our results with experiments, as well as make reference to physical cases where the cone is rotating 'quickly' and the axial flow is increased from a zero value (ie. s decreasing from ∞), for example during the take-off phase of an aeroplane, once the rotating turbofans have reached an optimum rotational velocity.

Essentially, the boundary-layer flow undergoes competition between the streamwise flow component, due to the oncoming flow, and the rotational flow component, due to effect of the spinning cone surface, which can be described mathematically in terms of the control parameters, ψ and s. We present the results of convective instability analyses for the boundary-layer flow over broad and slender rotating cones in a variety of imposed axial flows, based on large Reynolds-number and short-wavelength asymptotics, as well as numerical solutions obtained via an Orr–Sommerfeld stability analysis.

References

[1] L. Rosenhead. *Laminar Boundary Layers.* Oxford, 1963.

[2] R. Kobayashi. Linear stability theory of boundary layer along a cone rotating in axial flow. *Bull. Japan Soc. Mech. Engrs.* 4:934–940, 1981.

[3] R. Kobayashi, Y. Kohama and M. Kurosawa. Boundary-layer transition on a rotating cone in axial flow. *J. Fluid Mech.* 127:341–52, 1983.

[4] S. J. Garrett, Z. Hussain and S. O. Stephen. Boundary-layer transition on broad cones rotating in an imposed axial flow. *AIAA J.* 48, No. 6:1184–1194, 2010.